# Approximation and Existence of Vacuum States in the Multi-scale Flows of Compressible Euler Equations.

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## Abstract

In this paper, we study the approximation and existence of vacuum states in the multi-scale gas flows governed by the Cauchy problem of compressible Euler equations containing a small parameter \eta in the initial density. The system of Euler equations is reduced to a hyperbolic resonant system at the vacuum so that the weak solution of the Riemann problem is not suitable as the building block of Glimm (or Godunov) scheme to establish the existence of weak solutions with vacuum states. We construct a new type of approximate solutions, which are the weak solutions to the regularized Riemann problem of the leading order system by the asymptotic expansion around vacuum states. Such approximate solution is obtained by solving the pressureless Euler equations with generalized Riemann data, it consists of constant states separated by a composite hyperbolic wave, which is a combination of one nonlinear hyperbolic waves and two discontinuous linear waves. We show the stability of such regularized Riemann solution, together with the numerical simulations, under the small perturbations of generalized Riemann data. Adopting such solution as the building block of generalized Glimm scheme, we establish the existence of the vacuum solutions by showing the stability and consistency of the scheme. The numerical simulation indicates that, for any small time t, the approximate solutions converges to the exact solutions of the Cauchy problem in L^1 as \eta approaches 0. The theoretical proof of L^1 convergence is also provided. The results of this paper can be applied to some hyperbolic resonant systems of balance laws. This is a joint work with Chia-Chieh Jay Chu (NTHU), Hsin-Yi Lee (NCU) and Ying-Chieh Lin (NUK).

# Revisit Abelian and Tauberian theorem for Logarithmic asymptotic behavior

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We revisit the relation between asymptotic behavior between large time and small frequency under Laplace transform. For logarithmic case, we have a simple proof for Abelian and Tauberian theorem. This result is a a special case of Karamata's general and highly technical theorem. Thanks to properties of logarithm, our proof only involves elementary real analysis and calculus. This talk is based on a joint work with Jhe-Kuan Su.

**Keywords**: Laplace transform, Tauberian theorem.

# A kinetic model for a polyatomic gas with temperature-dependent specific heats and its application to shock-wave structure

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The ellipsoidal statistical (ES) model of the Boltzmann equation for a polyatomic gas, proposed by Andries *et al.* [P. Andries et al., Eur. J. Mech. B/Fluids **19**, 813 (2000)], is extended to a polyatomic gas with temperature-dependent specific heats (thermally perfect gas). Then, the new model equation is used to investigate the structure of a plane shock wave with special interest in $CO_2$ gas, which is known to have a very large bulk viscosity, and in the case of relatively strong shock waves. The numerical and asymptotic analyses are performed in parallel to the paper by S. Kosuge and K. Aoki [S. Kosuge and K. Aoki, Phys. Rev. Fluids **3**, 023401 (2018)], where the structure of a shock wave in $CO_2$ gas was investigated using the ES model for a polyatomic gas with constant specific heats (calorically perfect gas). From the numerical and analytical results, the effect of temperature-dependent specific heats is clarified. This is a joint work with Shingo Kosuge and Kazuo Aoki.

**Keywords**: Boltzmann equation, ellipsoidal statistical model, polyatomic gas, temperature-dependent specific heats, shock-wave structure

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# On a new class of fractional partial differential equations

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In this work we continue to advance the theory regarding the Riesz fractional gradient in the calculus of variations and fractional partial differential equations begun in an earlierwork of the same name. In particular, we here establish an $L^1$ Hardy inequality, obtain further regularity results for solutions of certain fractional PDE, demonstrate the existence of minimizers for integral functionals of the fractional gradient with non-linear dependence in the field, and also establish the existence of solutions to corresponding EulerLagrange equations obtained as conditions of minimality.

**Keywords**: Fractional gradient, fractional Hardy inequality, fractional partial differential equations, interpolation, Dirichlet forms

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# The fractional Calderón problem

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## Abstract

We review recent progress in the fractional Calderón problem, where one tries to determine an unknown coefficient in a fractional Schrödinger equation from exterior measurements of solutions. This equation enjoys remarkable uniqueness and approximation properties, which turn out to yield strong results in related inverse problems.

# On the Uniqueness and Structure of Solutions to the System Arising from Maxwell-Chern-Simons $O(3)$ Sigma Model

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In this talk, we will talk about the uniqueness of topological multivortex solutions for the self-dual Maxwell-Chern-Simons $O(3)$ sigma model with Chern-Simons coupling parameter sufficiently large and the charge of electron either sufficiently small or large. Besides, we also establish the sharp region of flux-pairs for the non-topological solutions and provide the classification of radial solutions of all types for single vortex point case. This is a joint work with Jann-Long Chern.

**Keywords**: uniqueness, topological multivortex solution, Maxwell-Chern-Simons

# Debye layer in Poisson-Boltzmann model with isolated singularities

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In this talk, we will show the existence of solutions to the charge-conserving Poisson-Boltzmann equation with Dirichlet boundary condition on $\partial\Omega$. Here $\Omega$ is a smooth simply connected bounded domain in $\mathbb{R}^n$ with $n \geq 2$. When $n = 2$, the solutions can have isolated singularities at prescribed points in $\Omega$. As a small parameter $\epsilon$ tends to zero, the solutions develop Debye boundary layer near the boundary $\partial\Omega$. In the interior of $\Omega$, the solutions converge to a unique constant. The limiting constant is explicitly calculated in terms of a novel formula which depends only on the supplied Dirichlet data on $\partial\Omega$. In addition, we also give a quantitative description on the asymptotic behaviour of the solutions as $\epsilon \to 0$. This is a joint work with Yong Yu (CUHK).

**Keywords**: Poisson-Boltzmann equation, boundary layer, asymptotic behaviour

# N-barrier maximum principle for reaction-diffusion equations

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This talk is devoted to the *N-barrier maximum principle* (NMBP) for the following $n$ equations

$$d_i(u_i)_{xx} + \theta(u_i)_x + u_i^{l_i} f_i(u_1, u_2, \cdots, u_n) = 0, \quad x \in \mathbb{R}, \quad i = 1, 2, \cdots, n, \tag{1}$$

where $u_i = u_i(x)$, $d_i$, $l_i > 0$, $\theta \in \mathbb{R}$, and $f_i(u_1, u_2, \cdots, u_n) \in C^0(\mathbb{R}^+ \times \mathbb{R}^+ \times \cdots \times \mathbb{R}^+)$ for $i = 1, 2, \cdots, n$. (1) arise from the study of traveling waves solutions of reaction-diffusion equations. For a solution $u_i(x)$ $(i = 1, 2, \cdots, n)$ of (1), the NBMP asserts that lower and upper bounds of $\sum_{i=1}^{n} \alpha_i u_i(x)$ can be given in terms of the parameters in (1) for arbitrary $\alpha_i$ $(i = 1, 2, \cdots, n)$. We will introduce briefly our work as follows:

- **Linear diffusion**: We prove the NBMP for (1) in [3]. In particular, the case $n = 2$ is proved in [2]. Under more restricted conditions on the parameters in (1) with $n = 3$, we give the lower and upper bounds of $u_1 + u_2 + u_3$ in [1].
- **Nonlinear diffusion**: With $(u_i)_{xx}$ replaced by $(u_i^2)_{xx}$, the NBMP is established for $n = 2$ in [4].
- **Discretized diffusion**: Recently, we have generalized the NBMP for a discretized diffusion version of (1) when $n = 2$ in [5].

Finally, as an application we use our NBMP to show the nonexistence of a three-species system.

**Keywords**: maximum principle, traveling wave solutions, reaction-diffusion equations.

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# On a class of indefinite nonlinear Schrödinger-Poisson system with steep potential well

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In this talk, we are concerned with the Schrödinger-Poisson systems with steep potential well and sign-changing weight functions. More precisely, the nonlinearity is a combination of a linear term and a superlinear term in the form $\lambda f(x)u + g(x)|u|^{p-2}u$, where $\lambda > 0, 4 \leq p < 6$ and $f$ and $g$ are allowed to be sign-changing. It is well known that a class of Schrödinger-Poisson systems with the above nonlinearity and the potential being positive constant has two positive solutions when the limit of infinity $\lim_{|x|\to\infty} g(x) = g_\infty < 0$ and either $K = 0$ a.e. in the set $\{x \in \mathbb{R}^3 : g(x) = 0\}$ or $\int_{\mathbb{R}^3}(ge_1^4 - K\phi_{e_1}e_1^2)dx < 0$ holds, see Huang-Rocha-Chen, J. Differential Equations 255 (2013); Chen, Nonlinear Anal. 21 (2015); Shen-Han, J. Math. Anal. Appl. 426 (2015). The main purpose is to obtain the existence and multiplicity of positive solutions without the above assumptions for $g$ and $K$. The results are obtained via variational method. This is a joint work with Prof. Tsung-fang Wu.

**Keywords**: Non-autonomous Schrödinger-Poisson system, indefinite nonlinearity, steep potential well, variational methods.

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